Effects of QCD bound states on relic abundance

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(based on...) **work in progress with F. Luo (IPMU)**

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Consider colored particle with mass

$m \gtrsim 1\mathrm{TeV} \gg \Lambda_{\mathrm{QCD}}$ in the early universe

Coulomb potential	$V \sim \frac{\alpha_s}{r}$
Binding energy	$E_B \sim \alpha_s^2 m \gtrsim 10\mathrm{GeV}$
inverse Bohr radius	$a^{-1} \sim \alpha_s m \gtrsim 100\mathrm{GeV}$

we consider (perturbatively) QCD bound state way before QCD phase transition occurs, and its interaction with dark matter.

As an example, consider R-parity conserving Minimal Supersymmetric Standard Model (MSSM)

Consider the R-odd lightest SUSY particle (LSP) as the lightest neutralino χ_1 and is the dark matter.

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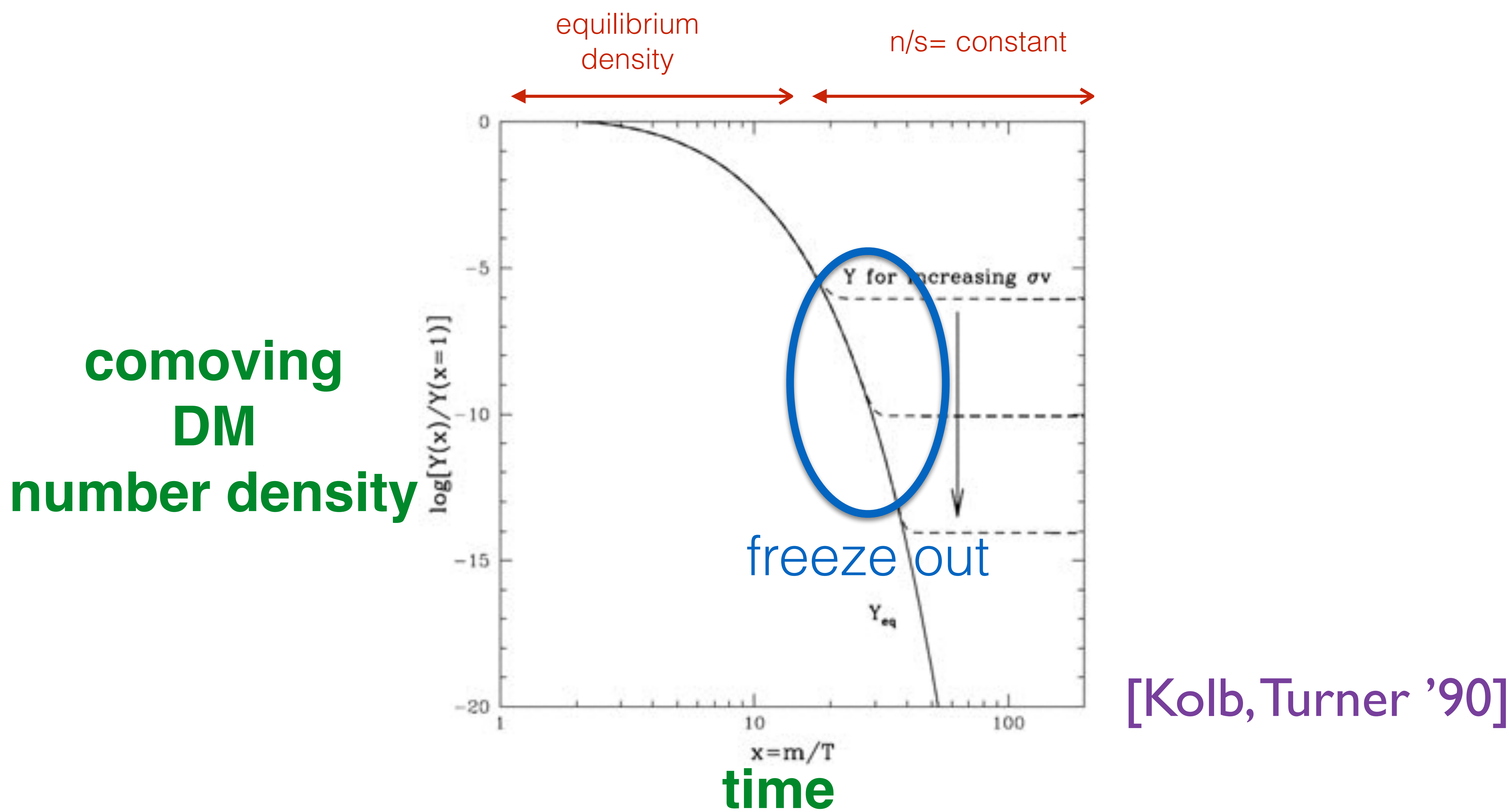
Consider χ_1 produced thermally.

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle \sigma v \rangle_{11} (n_1^2 - n_1^{eq2})$$

Standard DM relic abundance calculation



larger annihilation cross section -> smaller relic abundance

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Wino-like neutralino: ~3 TeV

Higgsino-like neutralino: ~1 TeV

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Bino?

depends on the masses of squarks & sleptons

usually bino is overproduced if sfermions are heavy

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Consider the R-odd lightest SUSY particle (LSP) as the lightest neutralino χ_1 and is the dark matter.

Consider χ_1 produced thermally.

Specifically, consider LSP coannihilating with an almost mass-degenerate R-odd SUSY particle χ_2 (not necessarily the second lightest neutralino). **Coannihilation** becomes vital.

How coannihilation works?

[Griest, Seckel '91]

conditions:

χ_2 has large annihilation cross section with *itself* or χ_1

$$\chi_2\chi_2 \leftrightarrow SMSM \qquad \chi_2\chi_1 \leftrightarrow SMSM$$

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$$\chi_2\chi_2 \leftrightarrow SMSM \qquad \chi_2\chi_1 \leftrightarrow SMSM$$

χ_2 can convert to χ_1 efficiently.

$$\chi_2 SM \leftrightarrow \chi_1 SM$$

Boltzmann equations

For simplicity, consider

$$\frac{dn_1}{dt} + 2Hn_1 = -\langle\sigma v\rangle_{11}(n_1^2 - n_{1eq}^2)$$

$$\frac{dn_2}{dt} + 3Hn_2 = -\langle\sigma v\rangle_{22}(n_2^2 - n_2^{eq2})$$

fast conversion means that

$$n_2/n_1 = n_2^{eq}/n_1^{eq} = \frac{g_2 m_2^{3/2}}{g_1 m_1^{3/2}}\exp(-(m_2 - m_1)/T)$$

note that $n_i^{eq} = g_i\,(m_i T/2\pi)^{3/2}\,e^{-m_i/T}$

Boltzmann equations

assuming fast conversion $\chi_2 SM \leftrightarrow \chi_1 SM$

defining $n \equiv n_1 + n_2$

$$\frac{dn}{dt} + 3Hn = -\boxed{\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2}} \left(n^2 - n_{eq}^2\right)$$

call this $\langle \sigma v \rangle_{\rm eff}$

compare with $$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM} \left(n_\chi^2 - {n_\chi^{eq}}^2\right)$$

without coannihilation

$$\frac{dn}{dt} + 3Hn = -\boxed{\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2}} (n^2 - n_{eq}^2)$$

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Two limits

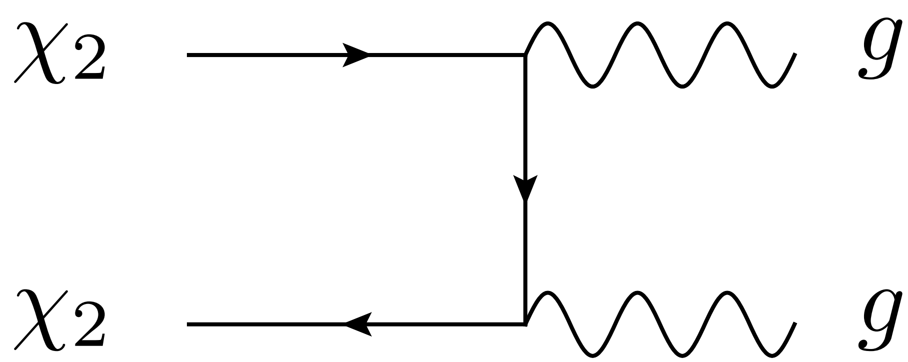
$\boxed{m_2 \gg m_1}$: $\langle \sigma v \rangle_{\text{eff}} \simeq \langle \sigma v \rangle_{11 \to SM}$

$\boxed{m_2 = m_1}$: $\langle \sigma v \rangle_{\text{eff}} = \dfrac{g_1^2 \langle \sigma v \rangle_{11 \to SM} + g_2^2 \langle \sigma v \rangle_{22 \to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12 \to SM}}{(g_1 + g_2)^2}$

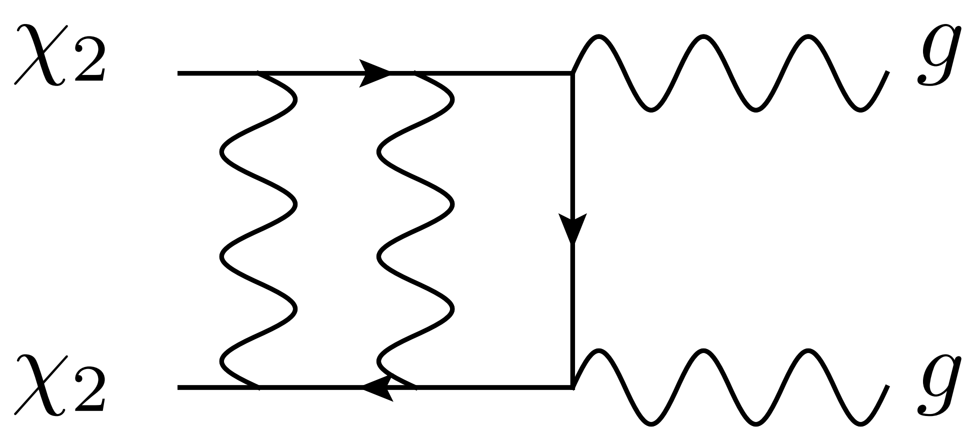
note that $n_i^{eq} = g_i \, (m_i T / 2\pi)^{3/2} \, e^{-m_i/T}$

We consider dark matter accompanied by an almost mass-degenerate colored particle.

If χ_2 is colored (squark or gluino in MSSM)
QCD Sommerfeld effect is important



tree-level annihilation



non-perturbative (Sommerfeld) effect that modifies the initial-state wave function

see e.g. [De Simone et al.'14]

If χ_2 is colored (squark or gluino in MSSM) formation of QCD bound state of χ_2 could be important as well

$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g,\ \tilde{R} \leftrightarrow gg$ for gluino [Ellis et al.'15]

$\tilde{t}\tilde{t} \leftrightarrow \tilde{\eta}g,\ \tilde{\eta} \leftrightarrow gg$ for stop

Compare recombination process $e^- p \leftrightarrow H\gamma$

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Compare recombination process $e^- p \leftrightarrow H\gamma$

note: bound state formation is important only when

$$\Gamma_{\rm ann} \gtrsim \Gamma_{\tilde{t}/\tilde{g}}$$

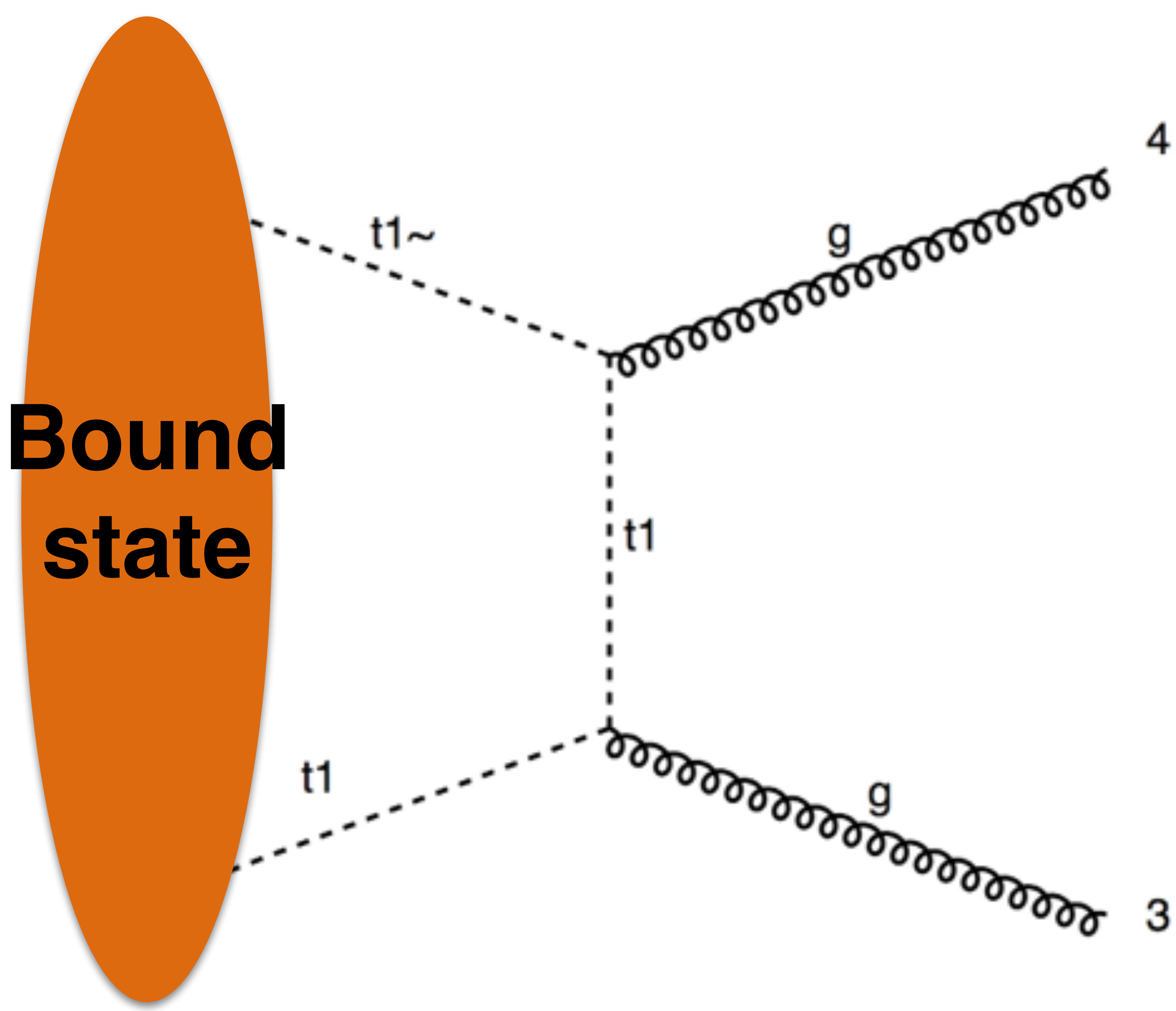
bound state annihilation rate

decay rate

note that bound state annihilation removes 2 R-odd particles, thus helps reducing DM density

gluino bound state $\tilde{R} \leftrightarrow gg$

stop bound state $\tilde{\eta} \leftrightarrow gg$



call the colored particle X and bound state η

one needs to solve the coupled Boltzmann eq. including the bound state η

$$\frac{dn_\eta}{dt} + 3Hn_\eta = -\Gamma_\eta (n_\eta - n_\eta^{eq}) + \Gamma_{bsf} (n_X^2 - n_X^{eq2} \frac{n_\eta}{n_\eta^{eq}})$$

bound state annihilation rate

bound state formation rate

bound state dissociation rate

Solving the coupled Boltzmann equations

$$\frac{dn_1}{dt} + 2Hn_1 = -\langle \sigma v \rangle_{11}(n_1^2 - n_{1eq}^2)$$

$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma v \rangle_{XX}(n_X^2 - n_X^{eq2}) - \Gamma_{bsf}(n_X^2 - n_X^{eq2}\frac{n_\eta}{n_\eta^{eq}})$$

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bound state number density is exponentially suppressed.
One can set LHS to zero as an approximation.
(the validity of this approx. has been checked numerically)

Solving the coupled Boltzmann equations

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Then, the Boltzmann equation is modified by adding the following terms:

$$\frac{dn}{dt} + 3Hn \simeq -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij\to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left(n^2 - n_{eq}^2\right)$$

$$\boxed{-\langle \sigma v \rangle_{XX\to\eta g} \frac{\langle \Gamma \rangle_{\eta\to gg}}{\langle \Gamma \rangle_{\eta\to gg} + \langle \Gamma \rangle_{\eta g\to XX}} (n_X^2 - n_X^{eq2})}$$

bound state formation rate

bound state annihilation rate

bound state dissociation rate

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because gluon is not energetic enough to dissociate the bound state

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(at temperature T < binding energy)

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late-time "annihilation" is important! One needs to solve the Boltzmann eqs. numerically

Calculation of bound state formation/dissociation rate

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Use Coulomb approximation to describe the bound state

$$V(r) = -C\frac{\alpha_s}{r}$$

with $$C = \frac{1}{2}\left(C_1 + C_2 - C_{(12)}\right)$$

SU(3) quadratic casimir of constituent particle

SU(3) quadratic casimir of bound state

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MSSM	binding	non-binding
$\tilde{g}\tilde{g}$	$\mathbf{1}, \mathbf{8_S}, \mathbf{8_A}$	$\mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$
$\tilde{t}\tilde{t}^*$	$\mathbf{1}$	$\mathbf{8}$
$\tilde{t}\tilde{t}$	$\overline{\mathbf{3}}$	$\mathbf{6}$
$\tilde{t}\tilde{g}$	$\mathbf{3}, \overline{\mathbf{6}}$	$\mathbf{15}$

MSSM	SU(3)	C
$(\tilde{g}\tilde{g})$	$\mathbf{1}$	3
	$\mathbf{8}$	$3/2$
$(\tilde{t}\tilde{t}^*)$	$\mathbf{1}$	$4/3$
$(\tilde{t}\tilde{t}), (\tilde{t}^*\tilde{t}^*)$	$\overline{\mathbf{3}}, \mathbf{3}$	$2/3$
$(\tilde{t}\tilde{g}), (\tilde{t}^*\tilde{g})$	$\mathbf{3}, \overline{\mathbf{3}}$	$3/2$
	$\overline{\mathbf{6}}, \mathbf{6}$	$1/2$

Consider photoelectric effect as an analogy

photoelectric effect: $H\gamma \to e^- p$

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Electromagnetic Hamiltonian $H = \dfrac{1}{2m}(\vec{p} + e\vec{A})^2$

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calculate the matrix $\langle \phi_f | \frac{e}{m}\vec{A}\cdot\vec{p} | \phi_i \rangle$

free particle wave function

bound state wave function

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rescale with appropriate color factors

Bound state formation rate is related to the dissociation rate via the Milne relation (or principle of detailed balance)

$$n_{X_1}^{eq} n_{X_2}^{eq} \sigma_{bsf} v_{rel} \left(1 + \frac{1}{e^{\omega/T} - 1}\right) f(v_{rel}) dv_{rel} = n_{\eta}^{eq} \sigma_{dis} dn_g^{eq}$$

bound state
formation rate

bound state
dissociation rate

scalar triplet bound state (Stoponium)

$$\tilde{t}\tilde{t} \to g\eta_{\tilde{t}}$$

we consider only the ground state

$$\begin{aligned}
E_B &= \left(\frac{4}{3}\alpha_s\right)^2 \left(\frac{m_{\tilde{t}}}{2}\right)/2\,, \\
a^{-1} &= \left(\frac{4}{3}\alpha_s\right)\left(\frac{m_{\tilde{t}}}{2}\right)\,, \\
\nu &= \left(\frac{1}{6}\alpha_s\right)/v_{rel}\,,
\end{aligned}$$

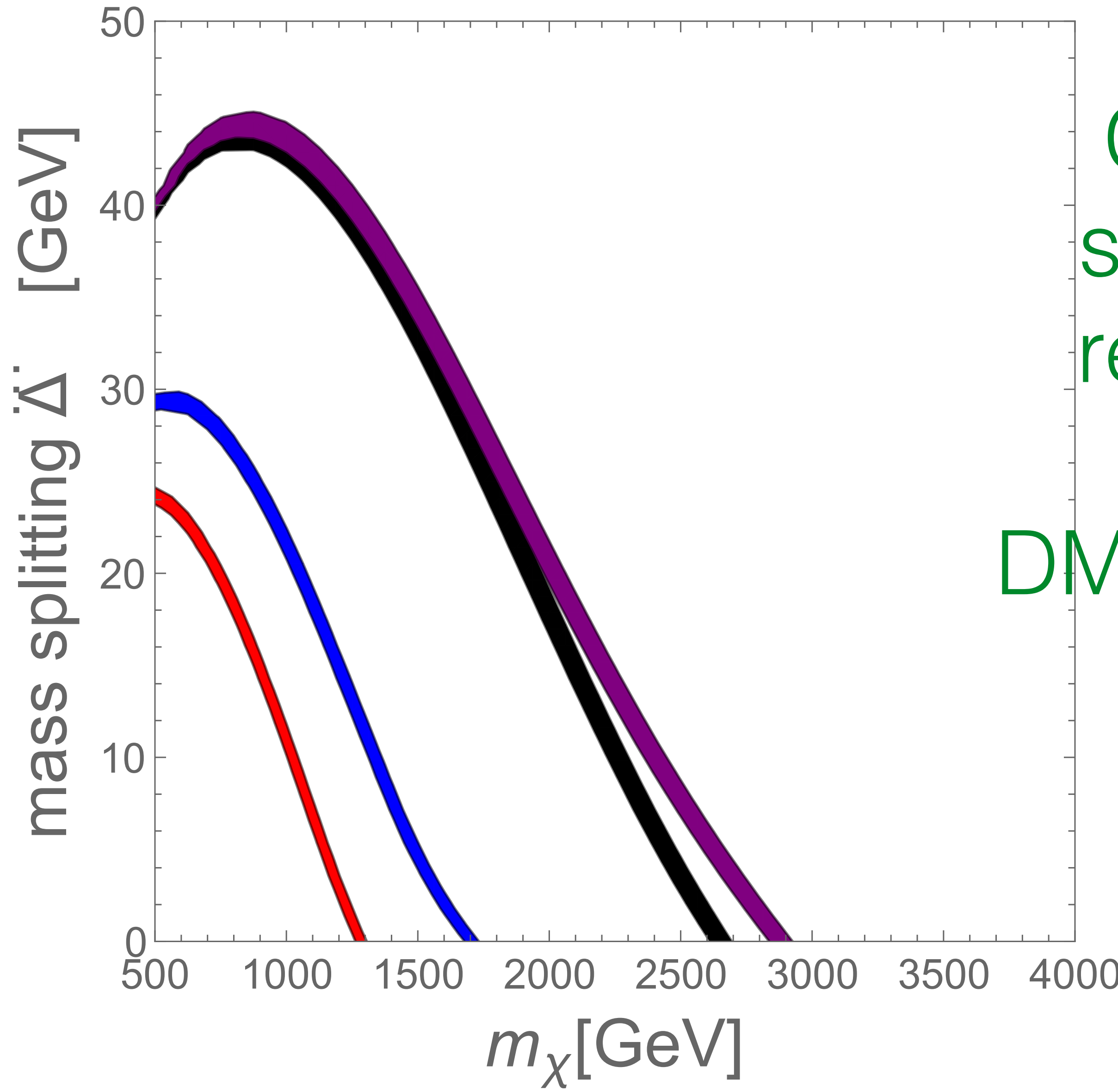
$$\begin{aligned}
\sigma^0_{dis} &= \frac{2^6\pi^2}{3}\alpha_s a^2 \left(\frac{E_B}{\omega}\right)^4 \frac{1+\nu^2}{1+(8\nu)^2}\frac{e^{4\nu\cot^{-1}(8\nu)-2\pi\nu}}{1-e^{-2\pi\nu}}\,, \\
\sigma_{dis} &= \frac{4}{3}\times\frac{1}{8}\times\sigma^0_{dis} \\
\sigma_{rec} &= \frac{4}{9}\left(\frac{4}{3}\alpha_s\right)^2\left(1+(8\nu)^2\right)^2(8\nu)^{-2}\sigma_{dis}
\end{aligned}$$

dissociation rate

formation rate

Scalar triplet (stop) coannihilation

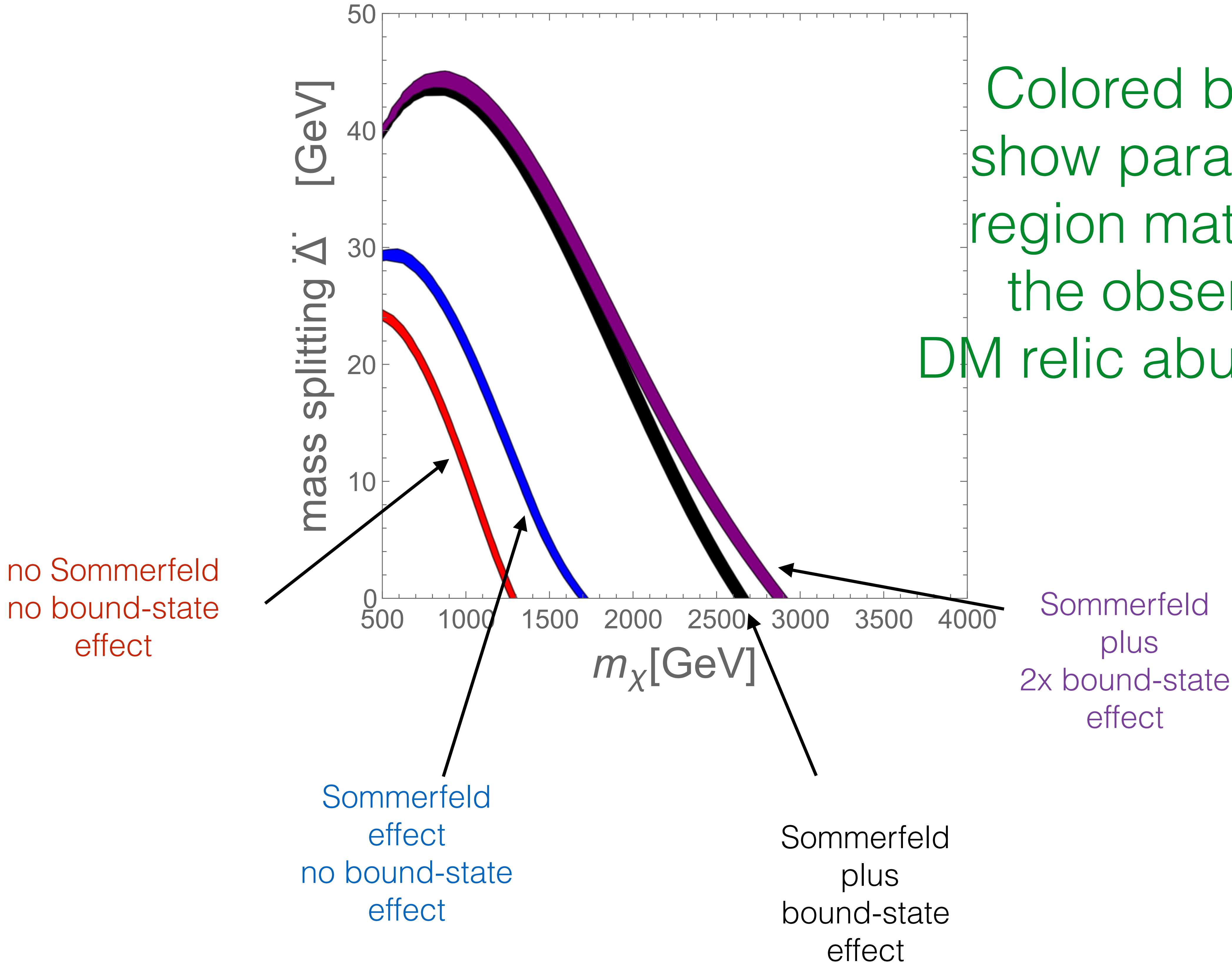
DM-stop mass splitting



Colored bands show parameter region matching the observed DM relic abundance

DM mass

Scalar triplet (stop) coannihilation



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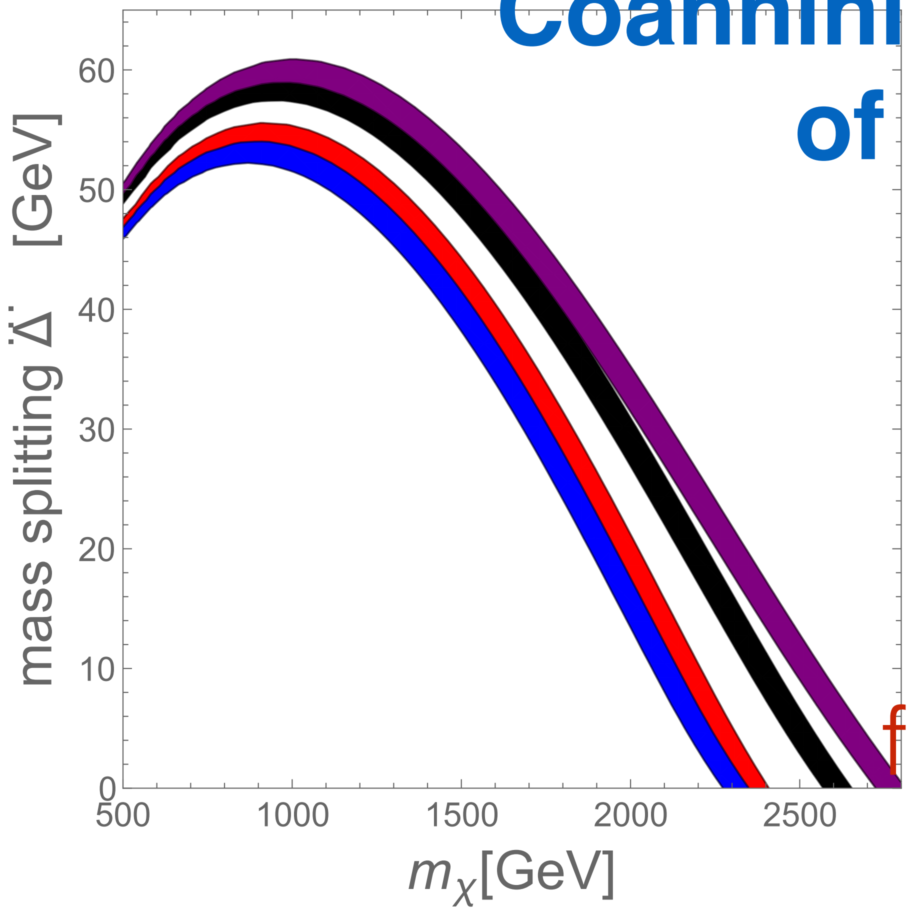
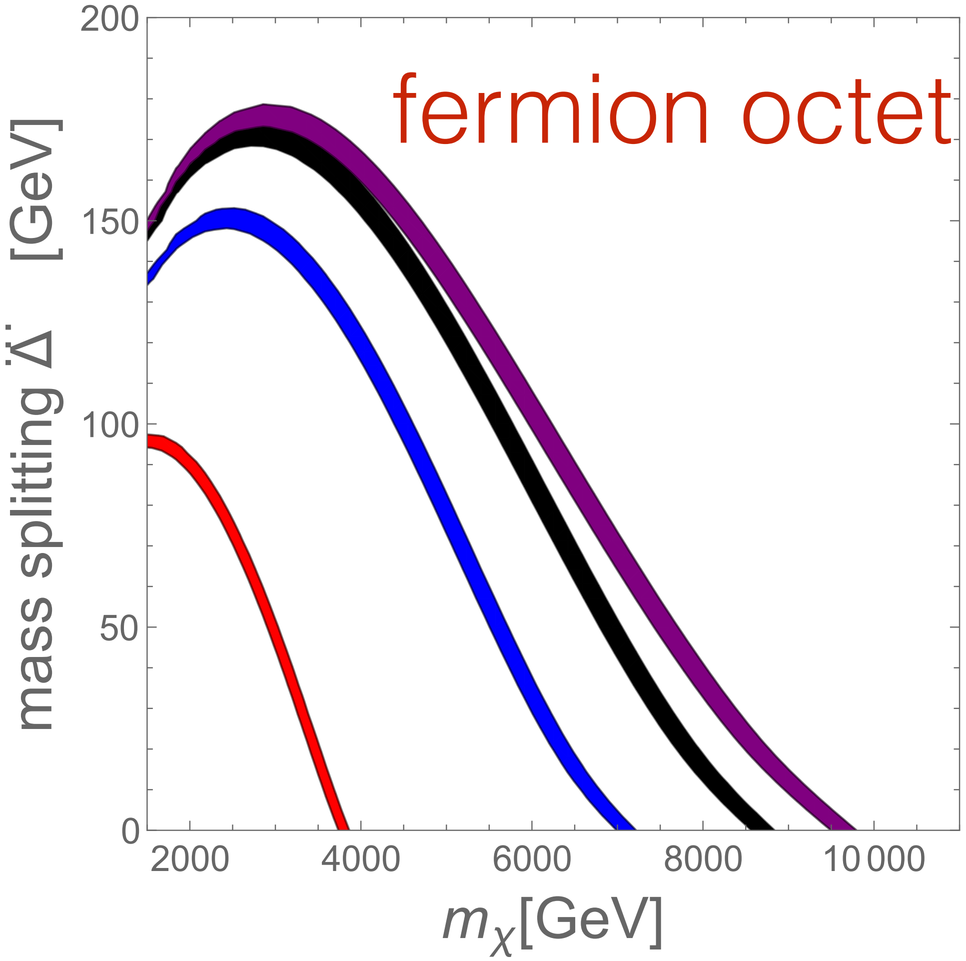
Coannihilation with other types of colored particle

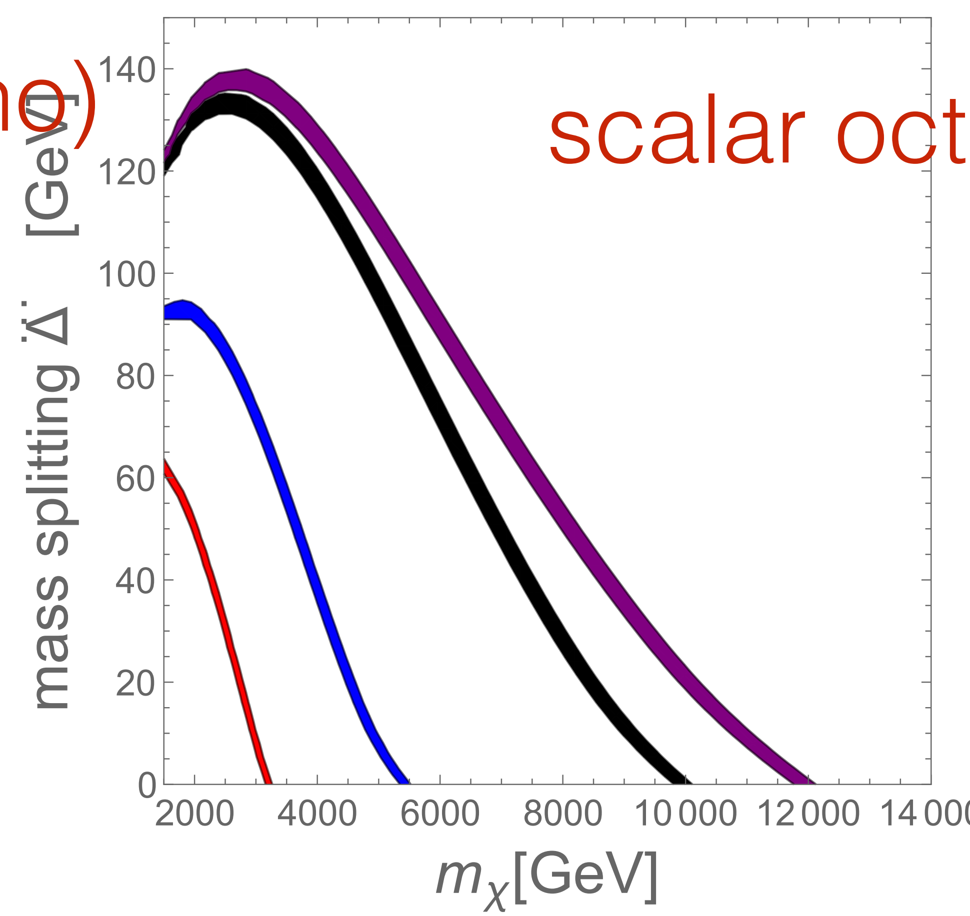
(fast conversion implicitly assumed)



fermion triplet

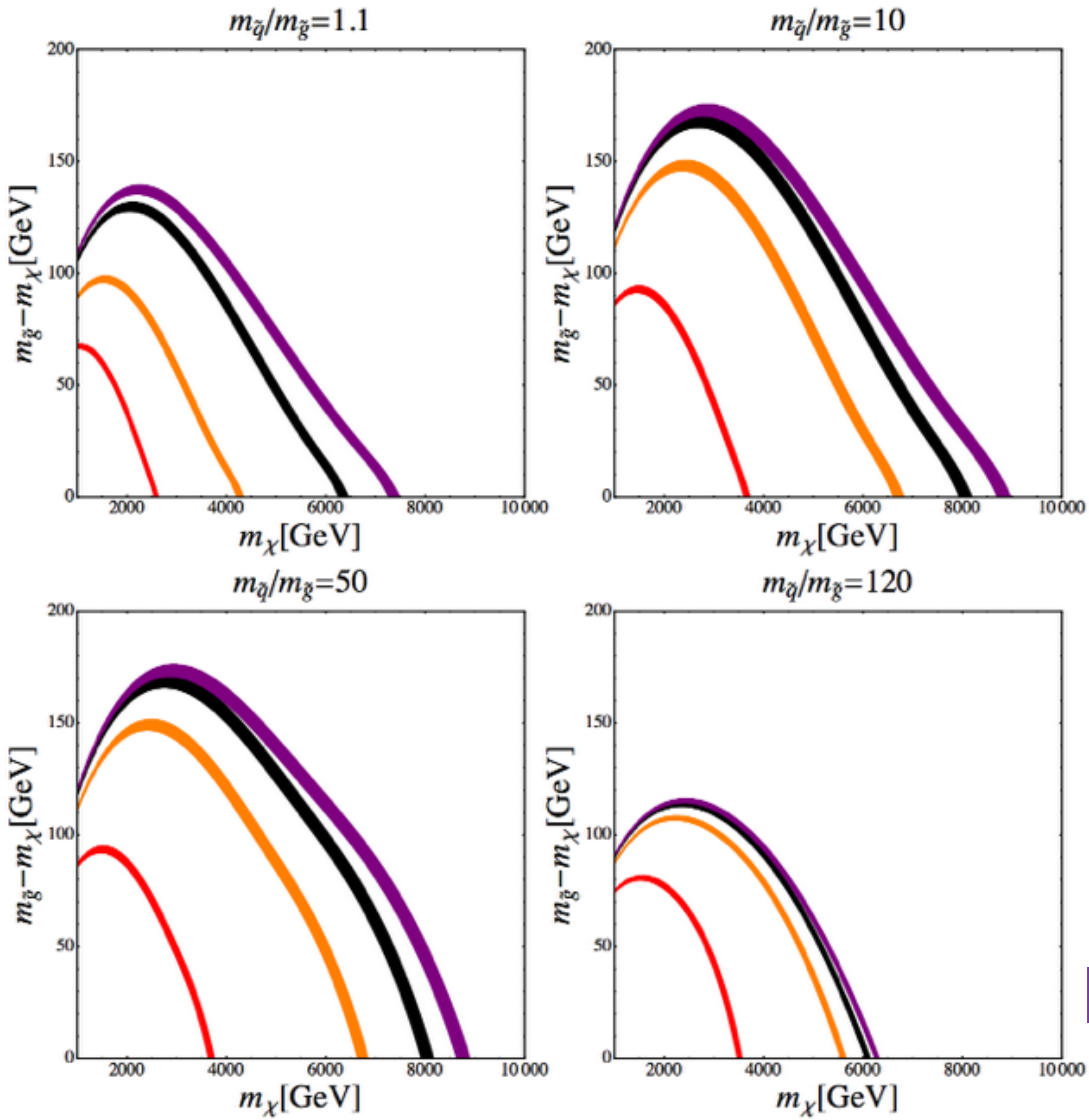
fermion octet (gluino)





scalar octet

gluino coannihilation
(with conversion taken into account appropriately)

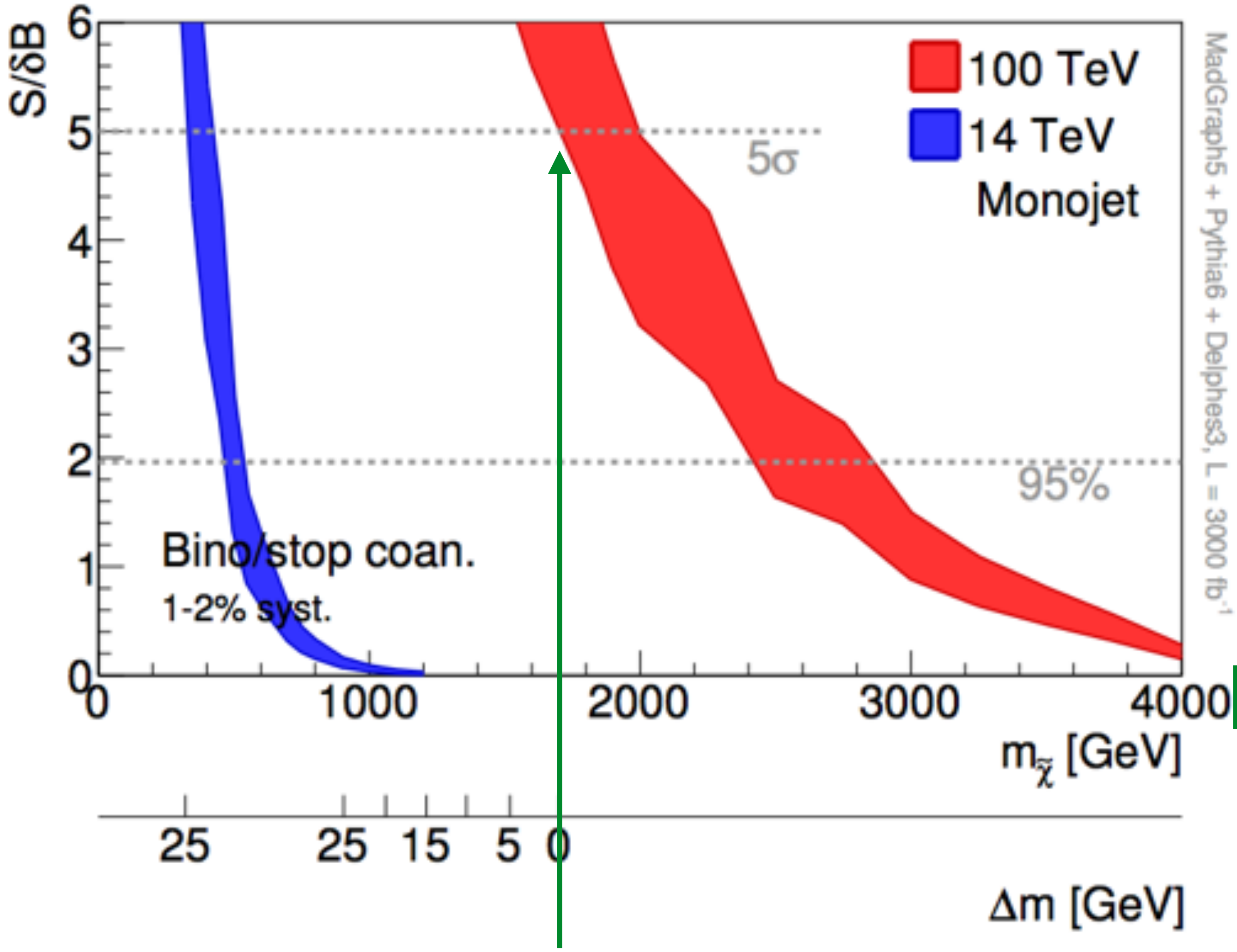


[Ellis et al.‘15]

a short comment on 100 TeV collider prospects

bino/stop coan. 5-sigma discovery becomes impossible at 100 TeV collider

significance

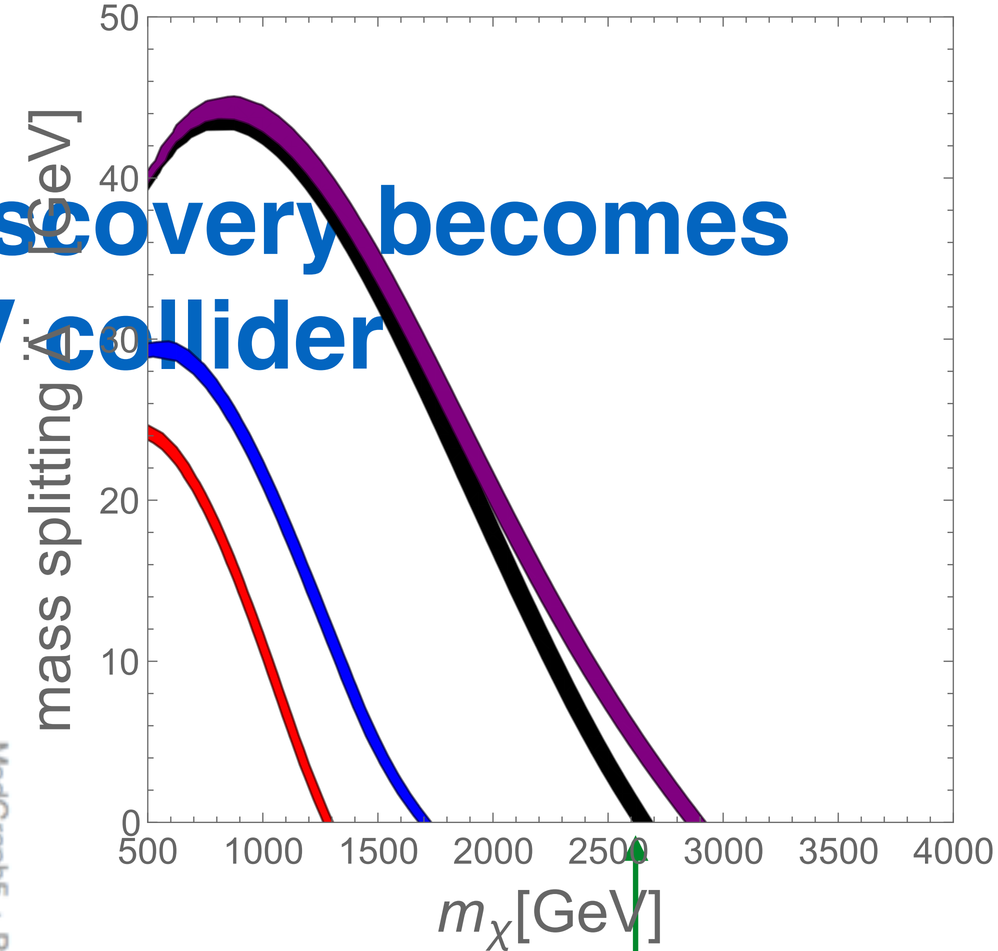


DM mass

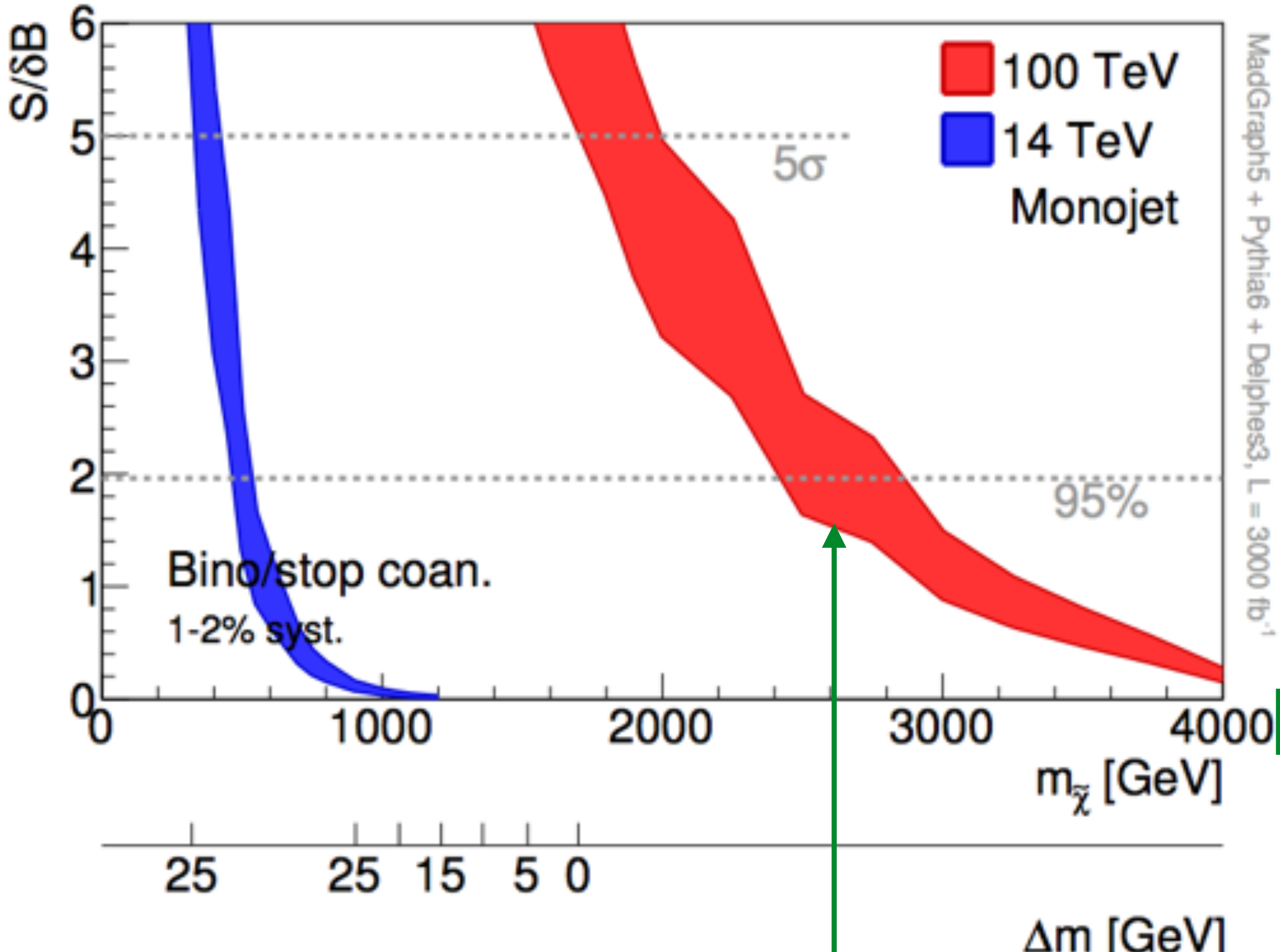
previous estimate

[Low, Wang '14]

bino/stop coan. 5-sigma discovery becomes impossible even at 100 TeV collider



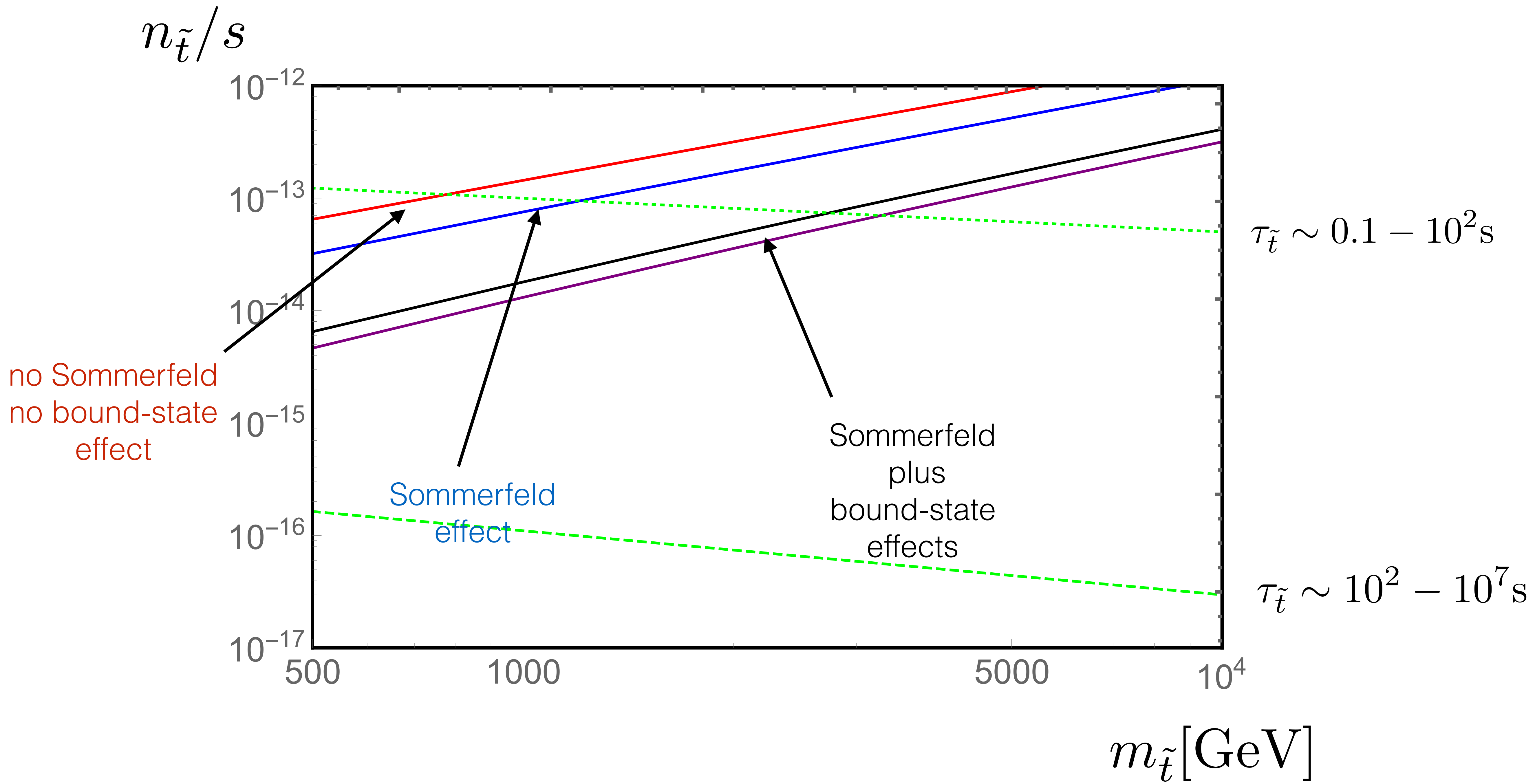
significance



DM mass

[Low, Wang '14]

Other implications of bound-state effects: BBN constraints on long-lived particles



see e.g. [Kawasaki et al '04]

Summary

We have considered dark matter accompanied by an almost mass-degenerate colored particle.

Bound state of the colored particles can increase the effective annihilation cross section significantly

Backup

How large are bound-state effects?

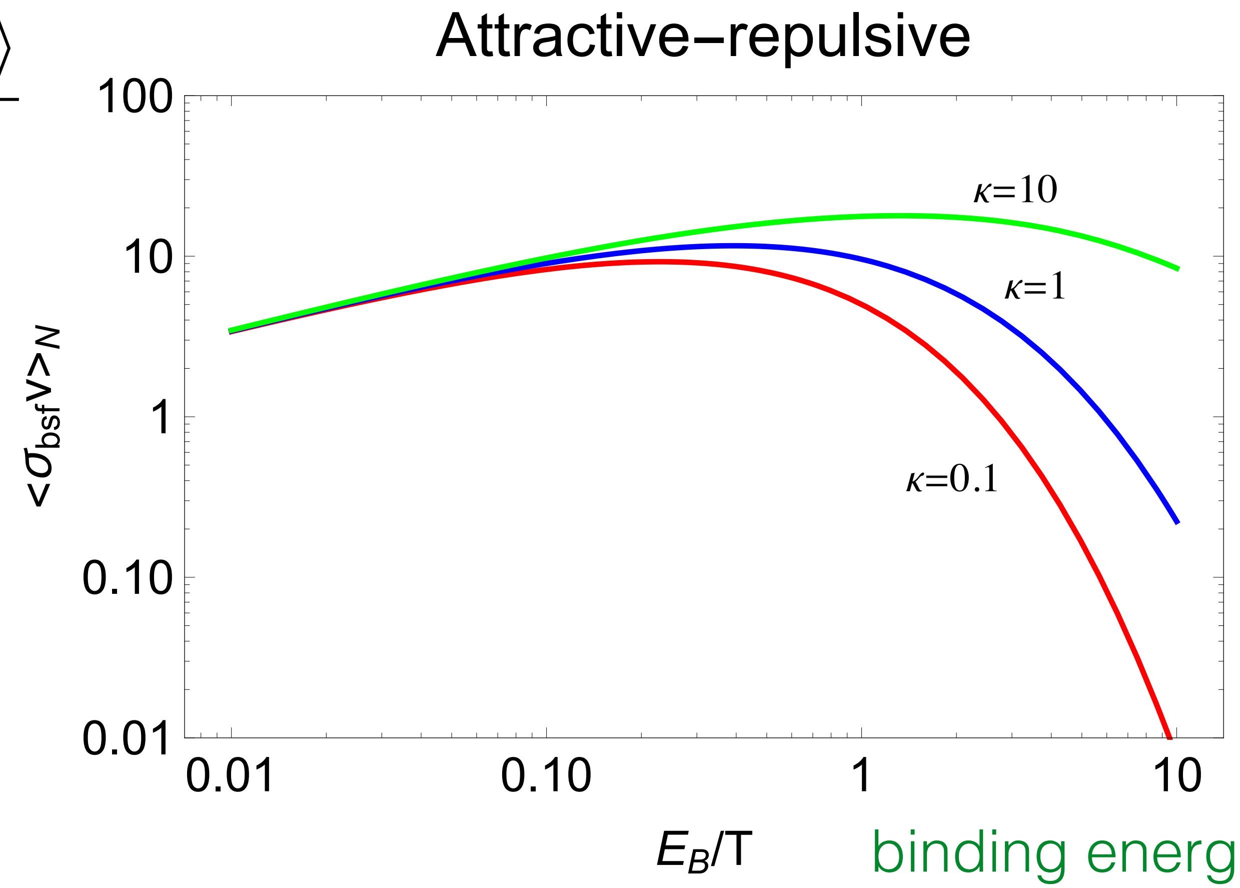
for gluino

$$\frac{\sigma_{bsf} v_{rel}}{S_{ann}(\sigma_{ann} v_{rel})} \sim 1.4 \qquad (v_{rel} \to 0)$$

for stop

$(\kappa = 8)$

$$\frac{\langle \sigma_{bsf} v_{rel} \rangle}{\sigma_{ann} v_{rel}}$$



binding energy/temperature